**A Taste of Curry’s Paradox**

**I.** Logical Consequence and Motivation

* Logical consequence in formal systems, one constituent: T-scheme
  + **T-Scheme**: φ ↔ Tr[φ]
  + Virtue: Captures intuitions about truth, i.e. what ‘true’ means

**II.** Ingredients of **C-Curry**

* T-Scheme coupled with **Diagonalization Sentence**: φ ↔ (Tr[φ] → ψ)…

1. φ Assume for Conditional Proof
2. φ → ψ 1, Substitution via Diagonalization Sentence
3. ψ 1,2 MP
4. φ → ψ 1-3 Discharge Conditional Assumption
5. φ 4, Substitution via Diagonalization Sentence
6. ψ 4,5, MP

* …*trivializes* the formal system.
  + Options: Removing Conditional Proof or Removing Contraction
    - **Conditional Proof**: If A├ B then ├ (A → B)
    - **Contraction**: If A, A ├ B then A├ B

**III.** Ingredients of **V-Curry**

* Logical consequence in formal systems, another constituent: V-Scheme
  + **V-Scheme**: α├ β ↔ Val[α, β]
  + Virtue: Captures intuitions about validity, i.e. what ‘follows from’ means
    - **VI**: If α├ β then ├ Val[α, β] (Validity Introduction ‘Rule’)
    - **VE**: α, Val[α, β]├ β (Validity Elimination ‘Rule’)
* VI, VE, coupled with the analogous Diagonalization Sentence: α ↔ Val[α, β]…

1. α Assume for VI
2. Val[α , β] 1, Substitution via Diagonalization Sentence
3. β 1,2 VE
4. α → β 1-3 Discharge VI Assumption
5. α 4, Substitution via Diagonalization Sentence
6. β 4,5, VE

* …*trivializes* the formal system.
  + Options Considered (Beall/Murzi): Removing VI or Removing Contraction
    - But, VI is part of the V-Schema (it *is* validity), reject Contraction!

**IV.** The Error

* V-Scheme parallels T-Scheme, is intuitive, and captures aspect of Logical Consequence
* However, V-Scheme is *not equivalent* to VI and VE (pace Beall/Murzi), recall -
  + V-Scheme: α├ β ↔ Val[α, β]
  + VI: If α├ β then ├ Val[α, β]
  + VE: α, Val[α, β]├ β
* However, for *equivalence*, we must change VI, VE, and V-Scheme, perhaps to…
  + V-Scheme\*: α├ β ↔ **α**, Val[α, β]
  + VI\*: If α├ β then ├ **α**, Val[α, β]
  + VE\*: **If** α, Val[α, β] **then α├ β**
* …or change VE to...
  + VE#: **If** Val[α, β] **then α ├ β**
    - However, either option precludes V-Curry (for the same reason, no way to discharge assumptions, only substitutions!)

**V.** Conclusion

* Beall/Murzi attempt to motivate rejecting Contraction by showing the high cost of rejecting the alternative (VI corresponding to Conditional Proof)
  + Yet, they mischaracterize the V-Scheme
  + Thus, the motivation is no motivation at all
* C-Curry is still on the menu, but V-Curry has been removed as logically unappetizing

**\*\*\*VI.** Supplement

* Despite the above, there are good reasons to reject Contraction
  + Avoids paradoxes other than C-Curry
  + Formal systems with Contraction Conditions (C1-C4) are susceptible to paradox

**C1**: A→B ├ A ϕ B **C3**: A ϕ (A ϕ B) ├ A ϕ B

**C2**: A, A ϕ B ├ B **C4**: If A├ B then ├ A ϕ B

1. A Assumption for C4

2. A ϕ B 1, Substitution with Diagonalization Sentence: A ↔ (A ϕ B)

3. B 1, C2

4. A ϕ B 1-3 Discharge C4

5. A 4, Substitution of Diagonalization Sentence: A ↔ (A ϕ B)

6. B 4,5 C2

* Despite the above, there may be good reasons *not* to reject Contraction
  + We may also have to abandon **Cantor’s Theorem**…